Global Journal of Engineering Science and Research Management

SOME ROUGH NEUTROSOPHIC SIMILARITY MEASURES AND THEIR APPLICATION TO MULTI ATTRIBUTE DECISION MAKING

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KEYWORDS: Cosine similarity measure Dice similarity measure, Jaccard similarity measure, neutrosophic sets, rough neutrosophic sets weighted Dice similarity measure.

ABSTRACT
This paper presents rough Dice and Jaccard similarity measures between rough neutrosophic sets. Pramanik and Mondal proposed cosine similarity measure and applied for medical diagnosis. In the present paper, we propose some basic operational relations and weighted rough Dice and Jaccard similarity measures and investigate some of their properties. Decision making under rough neutrosophic environment is more flexible and easy to deal with indeterminate and inconsistent information. Finally, the Dice and Jaccard similarity measures are applied to a medical diagnosis problem with rough neutrosophic information.

INTRODUCTION
Zadeh [1] introduced the degree of membership in 1965 and defined the fuzzy set in order to deal with uncertainty. Atanassov [2] introduced the degree of non-membership in 1986 and defined the intuitionistic fuzzy set. Smarandache [3, 4, 5, 6] introduced the degree of indeterminacy as independent component and defined the neutrosophic set. To use the concept of neutrosophic set in practical fields such as real scientific and engineering applications, Wang et al.[7] restricted the concept of neutrosophic set to single valued neutrosophic set since single value is an instance of set value.

Similarity measures play an important role in the analysis and research of medical diagnosis, pattern recognition, decision making, and clustering analysis in uncertain, indeterminate and inconsistent environment. Various similarity measures of SVNSs have been proposed and mainly applied them to decision making, pattern recognition, and clustering analysis. While the concept of neutrosophic sets is a powerful tool to deal with indeterminate and inconsistent data, the theory of rough neutrosophic sets [8, 9] is also a powerful mathematical tool to deal with incompleteness.

Majumdar and Samanta [10] introduced the similarity measures of SVNSs based on distances, a matching function, membership grades, and then proposed an entropy measure for a SVNS. Ye [11] proposed three vector similarity measures for simplified neutrosophic sets. Ye [12] proposed Dice similarity measure for single valued neutrosophic

ISSN 2349-4506
Impact Factor: 2.265

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multisets. Ye [13] proposed improved cosine similarity measure for single valued neutrosophic sets based on cosine function. Ye [14] proposed the similarity measures of SVNSs for multiple attribute group decision making method with completely unknown weights. Ye and Zhang [15] further proposed the similarity measures of SVNSs for decision making problems. Ye [16] proposed distance based similarity measures of SVNSs and applied it to clustering analysis. Biswas et al. [17] studied cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers. Pramanik and Mondal [18] proposed rough cosine similarity measure in rough neutrosophic environment. Mondal and Pramanik [19] proposed refined cotangent similarity measure in single valued neutrosophic environment. Mondal and Pramanik [20] further proposed cotangent similarity measure under rough neutrosophic environments. In this paper we propose some similarity measures namely Dice and Jaccard similarity measures in rough neutrosophic environment.

Rest of the paper is structured as follows: Section 2 presents neutrosophic and rough neutrosophic preliminaries. Section 3 is devoted to introduce rough Dice and Jaccard similarity measure for rough neutrosophic sets and studied some of its properties. Section 4 presents decision making based on rough Dice and Jaccard similarity measure. Section 5 presents the application of rough Dice and Jaccard similarity measures in medical diagnosis. Section 6 presents the concluding remarks and future scope of research.

NEUTROSOPHIC PRELIMINARIES

Definitions of neutrosophic Set [3, 4, 5, 6]

Smarandache [3] originally introduced the concept of a neutrosophic set from philosophical point of view. A neutrosophic set A in a universal set X is characterized by a truth membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, $F_A(x)$ in X are real standard or nonstandard subsets of $]^-0, 1^+[$. These functions satisfy the following two conditions.

1. $T_A(x): X \to]^-0, 1^+[$, $I_A(x): X \to]^-0, 1^+[$, and $F_A(x): X \to]^-0, 1^+[$.
2. $^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

However, the neutrosophic set is difficult to apply in practical applications. To deal with practical applications, Wang et al. [7] introduced single valued neutrosophic set (SVNS) as a subclass of the neutrosophic set.

Definitions related to single valued neutrosophic Set [7]

Definition1 [7]: Let X be a universal set. A SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity membership function $F_A(x)$. Then, a SVNS A can be denoted as $A = \{(x/(T_A(x), I_A(x), F_A(x))) : x \in X\}$. The sum of $T_A(x)$, $I_A(x)$, and $F_A(x)$ satisfies the condition $0 \le \sup T_A(x) + \sup F_A(x) + \sup I_A(x) \le 3,\ \forall x \in X$

Definition 2[7]**:** Complement: $A^c = \{(x/(F_A(x), 1 - I_A(x), T_A(x))) : x \in X\}$

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Definition 3[7]: Inclusion: $A \subseteq B$, iff $T_A(x) \le T_A(x)$, $I_A(x) \ge I_A(x)$, and $F_A(x) \ge F_A(x)$.

Definition 4[7]: Equality: $A = B$, iff $A \subseteq B$ and $A \supseteq B$

Definition 5 [7]: (Union): The union of two SVNSs A and B is a SVNS C, written as $C = A \cup B$ and defined as follows:

$$C = \{(x/(T_C(x), I_C(x), F_A(x))) : x \in X\}$$
$$T_C(x) = \max(T_A(x), T_B(x));$$
$$I_C(x) = \min(I_A(x), I_B(x));$$
$$F_C(x) = \min(F_A(x), F_B(x)) \ \forall\, x \in X$$

Definition 6[7]: (Intersection): The intersection of two SVNSs A and B is a SVNS E, written as $E = A \cap B$ and defined as follows:

$$E = \{(x/(T_E(x), I_E(x), F_E(x))) : x \in X\}.$$

Here,

$$T_E(x) = \min(T_A(x), T_B(x));$$
$$I_E(x) = \max(I_A(x), I_B(x));$$
$$F_E(x) = \max(F_A(x), F_B(x))$$
$$F_C(x) = \min(F_A(x), F_B(x)), \ \forall\, x \in x \text{ in } X$$

Rough Neutrosophic Sets [8, 9].

Let Z be a non-null set and R be an equivalence relation on Z. Let P be a neutrosophic set in Z with the membership function T_P, indeterminacy function I_P and non-membership function F_P. The lower and the upper approximations of P in the approximation (Z, R) denoted by $\underline{N}(P)$ and $\overline{N}(P)$ are respectively defined as follows:

$$\underline{N}(P) = \left\langle \begin{array}{l} < x, T_{\underline{N}(P)}(x), I_{\underline{N}(P)}(x), F_{\underline{N}(P)}(x) > / \\ z \in [x]_R, x \in Z \end{array} \right\rangle,$$

$$\overline{N}(P) = \left\langle \begin{array}{l} < x, T_{\overline{N}(P)}(x), I_{\overline{N}(P)}(x), F_{\overline{N}(P)}(x) > / \\ z \in [x]_R, x \in Z \end{array} \right\rangle \qquad (1)$$

Here, $T_{\underline{N}(P)}(x) = \wedge_z \in [x]_R\, T_P(z)$, $I_{\underline{N}(P)}(x) = \wedge_z \in [x]_R\, I_P(z)$, $F_{\underline{N}(P)}(x) = \wedge_z \in [x]_R\, F_P(z)$, $T_{\overline{N}(P)}(x) = \vee_z \in [x]_R\, T_P(z)$

$I_{\overline{N}(P)}(x) = \vee_z \in [x]_R\, T_P(z)$, $\quad F_{\overline{N}(P)}(x) = \vee_z \in [x]_R\, I_P(z)$

ISSN 2349-4506
Impact Factor: 2.265

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So, $0 \le \sup T_{\underline{N}(P)}(x) + \sup I_{\underline{N}(P)}(x) + \sup F_{\underline{N}(P)}(x) \le 3$
$0 \le \sup T_{\overline{N}(P)}(x) + \sup I_{\overline{N}(P)}(x) + \sup F_{\overline{N}(P)}(x) \le 3$

Here $\vee$ and $\wedge$ denote "max" and "min'' operators respectively, $T_P(z)$, $I_P(z)$ and $F_P(z)$ are the membership, indeterminacy and non-membership functions of z with respect to P. It is easy to see that $\underline{N}(P)$ and $\overline{N}(P)$ are two neutrosophic sets in Z.

Thus NS mapping $\underline{N}$, $\overline{N} : N(Z) \rightarrow N(Z)$ are, respectively, referred to as the lower and upper rough NS approximation operators, and the pair $(\underline{N}(P), \overline{N}(P))$ is called the rough neutrosophic set [8], [9] in (Z, R).

From the above definition, it is seen that $\underline{N}(P)$ and $\overline{N}(P)$ have constant membership on the equivalence classes of R if $\underline{N}(P) = \overline{N}(P)$; .e. $T_{\underline{N}(P)}(x) = T_{\overline{N}(P)}(x)$, $I_{\underline{N}(P)}(x) = I_{\overline{N}(P)}(x)$, $F_{\underline{N}(P)}(x) = F_{\overline{N}(P)}(x) \; \forall \; x \in x$ in X .

P is said to be a definable neutrosophic set in the approximation (Z, R). It can be easily proved that zero neutrosophic set (0_N) and unit neutrosophic sets (1_N) are definable neutrosophic sets.

Definition 2.3.1

If $N(P)$ = ($\underline{N}(P), \overline{N}(P)$) is a rough neutrosophic set in (Z, R) , the rough complement [8, 9] of $N(P)$ is the rough neutrosophic set denoted by $\sim N(P) = (\underline{N}(P)^c, \overline{N}(P)^c)$, where $\underline{N}(P)^c$, $\overline{N}(P)^c$ are the complements of neutrosophic sets of $\underline{N}(P), \overline{N}(P)$ respectively.

$$\underline{N}(P)^c = \left\langle \begin{array}{l} < x, T_{\underline{N}(P)}(x), 1 - I_{\underline{N}(P)}(x),\ F_{\underline{N}(P)}(x) > / \\ , x \in Z \end{array} \right\rangle, \text{ and}$$

$$\overline{N}(P)^c = \left\langle \begin{array}{l} < x, T_{\underline{N}(P)}(x), 1 - I_{\overline{N}(P)}(x),\ F_{\overline{N}(P)}(x) > / \\ , x \in Z \end{array} \right\rangle \qquad (2)$$

Definition 2.3.2

If $N(P_1)$ *and* $N(P_2)$ are the two rough neutrosophic sets of the neutrosophic set P respectively in Z, then the following definitions [8], [9] hold good:

ISSN 2349-4506
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$$N(P_1)=N(P_2) \Leftrightarrow \underline{N}(P_1)=\underline{N}(P_2) \wedge \overline{N}(P_1)=\overline{N}(P_2)$$
$$N(P_1)\subseteq N(P_2) \Leftrightarrow \underline{N}(P_1)\subseteq\underline{N}(P_2) \wedge \overline{N}(P_1)\subseteq\overline{N}(P_2)$$
$$N(P_1)\bigcup N(P_2)=<\underline{N}(P_1)\bigcup\underline{N}(P_2),\ \overline{N}(P_1)\bigcup\overline{N}(P_2)>$$
$$N(P_1)\bigcap N(P_2)=<\underline{N}(P_1)\bigcap\underline{N}(P_2),\ \overline{N}(P_1)\bigcap\overline{N}(P_2)>$$
$$N(P_1)+N(P_2)=<\underline{N}(P_1)+\underline{N}(P_2),\ \overline{N}(P_1)+\overline{N}(P_2)>$$
$$N(P_1)\ .N(P_2)=<\underline{N}(P_1)\ .\ \underline{N}(P_2),\ \overline{N}(P_1)\ .\overline{N}(P_2)>$$

If N, M, L are the rough neutrosophic sets in (Z, R), then the following proposition are stated from definitions.
Proposition 1 [8], [9]

1. $\sim N(\sim N)=N$
2. $N\bigcup M=M\bigcup N, M\bigcup N=N\bigcup M$
3. $(L\bigcup M)\bigcup N=L\bigcup(M\bigcup N)$,
   $(L\bigcap M)\bigcap N=L\bigcap(M\bigcap N)$
4. $(L\bigcup M)\bigcap N=(L\bigcup M)\bigcap(L\bigcup N)$,
   $(L\bigcap M)\bigcup N=(L\bigcap M)\bigcup(L\bigcap N)$

Proposition 2 [8], [9]
De Morgan‘s Laws are satisfied for rough neutrosophic sets
1. $\sim(N(P_1)\bigcup N(P_2))=(\sim N(P_1))\bigcap(\sim N(P_2))$
2. $\sim(N(P_1)\bigcap N(P_2))=(\sim N(P_1))\bigcup(\sim N(P_2))$

Proposition 3[8], [9]
If P_1 and P_2 are two neutrosophic sets in U such that $P_1\subseteq P_2, then\ N(P_1)\subseteq N(P_2)$
1. $N(P_1\bigcap P_2)\subseteq N(P_2)\bigcap N(P_2)$
2. $N(P_1\bigcup P_2)\supseteq N(P_2)\bigcup N(P_2)$

Proposition 4 [8], [9]
1. $\underline{N}(P)=\sim\overline{N}(\sim P)$
2. $\overline{N}(P)=\sim\underline{N}(\sim P)$
3. $\underline{N}(P)\subseteq\overline{N}(P)$

ISSN 2349-4506
Impact Factor: 2.265

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For the proofs of the above mentioned propositions see [8, 9].

SOME SIMILARITY MEASURES UNDER ROUGH NEUTROSOPHIC ENVIRONMENT

Dice similarity measure under rough neutrosophic environment

In this section, we propose the Dice similarity measure and the weighted Dice similarity measure for rough neutrosophic sets and investigate their properties.

Definition 3.1: Assume that A and B are two rough neutrosophic sets denoted by

$A=\left\langle\left(\underline{T}_A(x_i),\underline{I}_A(x_i),\underline{F}_A(x_i)\right),\left(\overline{T}_A(x_i),\overline{I}_A(x_i),\overline{F}_A(x_i)\right)\right\rangle$ and $B=\left\langle\left(\underline{T}_B(x_i),\underline{I}_B(x_i),\underline{F}_B(x_i)\right),\left(\overline{T}_B(x_i),\overline{I}_B(x_i),\overline{F}_B(x_i)\right)\right\rangle$

in $X = \{x_1, x_2, \ldots, x_n)$. A Dice similarity measure between two rough neutrosophic sets A and B is defined as follows:

$$DIC_{RNS}(A,B)=$$
$$\frac{1}{n}\sum_{i=1}^{n}\frac{2.\begin{Bmatrix}\delta T_A(x_i)\,\delta T_B(x_i)+\delta I_A(x_i)\,\delta I_B(x_i)\\+\delta F_A(x_i)\,\delta F_B(x_i)\end{Bmatrix}}{\begin{Bmatrix}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{Bmatrix}} \tag{3}$$

Proposition 5:

Let A and B be rough neutrosophic sets then

1. $0\le DIC_{RNS}(A,B)\le 1$
2. $DIC_{RNS}(A,B)=DIC_{RNS}(B,A)$
3. $D_{RNS}(A, B) = 1$, iff $A = B$
4. If C is a RNS in Y and $A\subset B\subset C$ then, $DIC_{RNS}(A, C) \le DIC_{RNS}(A, B)$, and $DIC_{RNS}(A, C) \le DIC_{RNS}(B, C)$

Proof:

1. It is obvious because all positive values of Dice function are within 0 and 1.
2. It is obvious that the proposition is true.
3. When $A = B$, then obviously $DIC_{RNS}(A, B) = 1$. On the other hand if $DIC_{RNS}(A, B) =1$ then, $\delta T_A(x_i)=\delta T_B(x_i)$, $\delta I_A(x_i)=\delta I_B(x_i)$, $\delta F_A(x_i)=\delta F_B(x_i)$ *ie,*

$\underline{T}_A(x_i)=\underline{T}_B(x_i)$, $\overline{T}_A(x_i)=\overline{T}_B(x_i)$, $\underline{I}_A(x_i)=\underline{I}_B(x_i)$, $\overline{I}_A(x_i)=\overline{I}_B(x_i)$, $\underline{F}_A(x_i)=\underline{F}_B(x_i)$, $\overline{F}_A(x_i)=\overline{F}_B(x_i)$

This implies that $A = B$.

ISSN 2349-4506
Impact Factor: 2.265

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4. If $A \subset B \subset C$ **then we can write** $\underline{T}_A(x_i) \le \underline{T}_B(x_i) \le \underline{T}_C(x_i)$, $\overline{T}_A(x_i) \le \overline{T}_B(x_i) \le \overline{T}_C(x_i)$, $\underline{I}_A(x_i) \ge \underline{I}_B(x_i) \ge \underline{I}_C(x_i)$, $\overline{I}_A(x_i) \ge \overline{I}_B(x_i) \ge \overline{I}_B(x_i)$, $\underline{F}_A(x_i) \ge \underline{F}_B(x_i) \ge \underline{F}_C(x_i)$, $\overline{F}_A(x_i) \ge \overline{F}_B(x_i) \ge \overline{F}_C(x_i)$.

Hence we can write $DIC_{RNS}(A, C) \le DIC_{RNS}(A, B)$, and $DIC_{RNS}(A, C) \le DIC_{RNS}(B, C)$.
If we consider the weights of each element x_i, a weighted rough Dice similarity measure between rough neutrosophic sets A and B can be defined as follows:

$$DIC_{WRNS}(A,B) = \frac{1}{n}\sum_{i=1}^{n} w_i \frac{2\begin{Bmatrix}\delta T_A(x_i)\,\delta T_B(x_i) + \delta I_A(x_i)\,\delta I_B(x_i)\\ +\,\delta F_A(x_i)\,\delta F_B(x_i)\end{Bmatrix}}{\begin{Bmatrix}\left\langle(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2\right\rangle\\ +\left\langle(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2\right\rangle\end{Bmatrix}} \tag{4}$$

$w_i \in [0,1]$, i = 1, 2,…, n and $\sum_{i=1}^{n} w_i = 1$. If we take $w_i = \frac{1}{n}$, i = 1, 2,…, n, then $DIC_{WRNS}(A, B) = DIC_{RNS}(A,B)$

The weighted rough Dice similarity measure (WRNS) between two rough neutrosophic sets A and B satisfies the following properties:

Proposition 6:
1. $0 \le DIC_{WRNS}(A,B) \le 1$
2. $DIC_{WRNS}(A,B) = DIC_{WRNS}(B,A)$
3. $DIC_{WRNS}(A, B) = 1$, iff $A = B$
4. If C is a WRNS in Y and $A \subset B \subset C$ then, $DIC_{WRNS}(A, C) \le DIC_{WRNS}(A, B)$, and $DIC_{WRNS}(A, C) \le DIC_{WRNS}(B, C)$

Proof:
The proofs of the above properties are similar to the proof of the proposition (5).

Jaccard similarity measure under rough neutrosophic environment

Assume that there are two rough neutrosophic sets $A = \left\langle \left(\underline{T}_A(x_i), \underline{I}_A(x_i), \underline{F}_A(x_i)\right), \left(\overline{T}_A(x_i), \overline{I}_A(x_i), \overline{F}_A(x_i)\right)\right\rangle$ and

 ISSN 2349-4506
Impact Factor: 2.265

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$B = \left\langle \left(\underline{T}_B(x_i), \underline{I}_B(x_i), \underline{F}_B(x_i)\right), \left(\overline{T}_B(x_i), \overline{I}_B(x_i), \overline{F}_B(x_i)\right)\right\rangle$ in $X = \{x_1, x_2, \ldots, x_n\}$. A Jaccard similarity measure between two rough neutrosophic sets A and B is dedfined as follows:

$$JAC_{RNS}(A,B) = \frac{1}{n}\sum_{i=1}^{n} \frac{\left\{\begin{array}{l}\delta T_A(x_i)\delta T_B(x_i) + \delta I_A(x_i)\delta I_B(x_i) \\ +\delta F_A(x_i)\delta F_B(x_i)\end{array}\right\}}{\left\{\begin{array}{l}\left[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2\right] \\ +\left[(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2\right] \\ -\left[\begin{array}{l}\delta T_A(x_i)\delta T_B(x_i) + \delta I_A(x_i)\delta I_B(x_i) \\ +\delta F_A(x_i)\delta F_B(x_i)\end{array}\right]\end{array}\right\}} \quad (5)$$

Proposition 7

Let A and B be rough neutrosophic sets then

1. $0 \le JAC_{RNS}(A,B) \le 1$
2. $JAC_{RNS}(A,B) = JAC_{RNS}(B,A)$
3. $JAC_{RNS}(A, B) = 1$, iff $A = B$
4. If C is a RNS in Y and $A \subset B \subset C$ then, $JAC_{RNS}(A, C) \le JAC_{RNS}(A, B)$, and $JAC_{RNS}(A, C) \le JAC_{RNS}(B, C)$

Proof:

1. It is obvious because all positive values of Jaccard function are within 0 and 1.
2. It is obvious that the proposition is true.
3. When $A = B$, then obviously $JAC_{RNS}(A, B) = 1$. On the other hand if $JAC_{RNS}(A, B) = 1$ then,

$\delta T_A(x_i) = \delta T_B(x_i)$, $\delta I_A(x_i) = \delta I_B(x_i)$, $\delta F_A(x_i) = \delta F_A(x_i)$ *ie,*

$\underline{T}_A(x_i) = \underline{T}_B(x_i)$, $\overline{T}_A(x_i) = \overline{T}_B(x_i)$, $\underline{I}_A(x_i) = \underline{I}_B(x_i)$, $\overline{I}_A(x_i) = \overline{I}_B(x_i)$, $\underline{F}_A(x_i) = \underline{F}_B(x_i)$, $\overline{F}_A(x_i) = \overline{F}_B(x_i)$

This implies that A = B.

4. If $A \subset B \subset C$, then we can write $\underline{T}_A(x_i) \le \underline{T}_B(x_i) \le \underline{T}_C(x_i)$, $\overline{T}_A(x_i) \le \overline{T}_B(x_i) \le \overline{T}_C(x_i)$, $\underline{I}_A(x_i) \ge \underline{I}_B(x_i) \ge \underline{I}_C(x_i)$, $\overline{I}_A(x_i) \ge \overline{I}_B(x_i) \ge \overline{I}_B(x_i)$, $\underline{F}_A(x_i) \ge \underline{F}_B(x_i) \ge \underline{F}_C(x_i)$, $\overline{F}_A(x_i) \ge \overline{F}_B(x_i) \ge \overline{F}_C(x_i)$.

Hence we can write $JAC_{RNS}(A, C) \le JAC_{RNS}(A, B)$, and $JAC_{RNS}(A, C) \le JAC_{RNS}(B, C)$.

 ISSN 2349-4506
Impact Factor: 2.265

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If we consider the weights of each element x_i, a weighted rough Jaccard similarity measure between two rough neutrosophic sets A and B can be defined as follows:

$$JAC_{WRNS}(A,B)=$$

$$\frac{1}{n}\sum_{i=1}^{n} w_i \frac{\left\{\begin{array}{l}\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\ +\delta F_A(x_i)\delta F_B(x_i)\end{array}\right\}}{\left\{\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\ +\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\\ -\left[\begin{array}{l}\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\ +\delta F_A(x_i)\delta F_B(x_i)\end{array}\right]\end{array}\right\}} \qquad (6)$$

$w_i \in [0,1]$, i = 1, 2,…, n and $\sum_{i=1}^{n} w_i = 1$. If we take $w_i = \frac{1}{n}$, i = 1, 2,…, n, then $JAC_{WRNS}(A, B) = JAC_{RNS}(A,B)$

The weighted rough Jaccard similarity measure between two rough neutrosophic sets A and B also satisfies the following properties:

Proposition 8

1. $0 \le JAC_{WRNS}(A,B) \le 1$
2. $JAC_{WRNS}(A,B) = JAC_{WRNS}(B,A)$
3. $D_{WRNS}(A, B) = 1$, iff $A = B$
4. If C is a WRNS in Y and $A \subset B \subset C$ then, $JAC_{WRNS}(A, C) \le JAC_{WRNS}(A, B)$, and $JAC_{WRNS}(A, C) \le JAC_{WRNS}(B, C)$

Proof:
Proofs of the above properties are similar to the proof of the proposition (6).

DECISION MAKING UNDER ROUGH NEUTROSOPHIC SETS BASED ON DICE AND JACCARD SIMILARITY MEASURES

Let $A_1, A_2, ..., A_m$ be a discrete set of candidates, $C_1, C_2, ..., C_n$ be the set of criteria, and $B_1, B_2, ..., B_k$ be the alternatives. The decision-maker provides the ranking of alternatives with respect to each candidate. Decision making procedure under rough neutrosophic sets based on Dice and Jaccard similarity measure is presented as following steps.

ISSN 2349-4506
Impact Factor: 2.265

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Step 1: Determination the relation between candidates and attributes

The ranking presents the performances of candidates A_i (i = 1, 2,..., m) against the criterion C_j (j = 1, 2, ..., n). The rough neutrosophic values associated with the candidates and their attributes for MADM problem are presented in the decision matrix (see the table 1).

Table 1: The relation between candidates and attributes

$$D = \left\langle \underline{d}_{ij}, \overline{d}_{ij} \right\rangle_{m\times n} = \begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline A_1 & \left\langle \underline{d}_{11}, \overline{d}_{11} \right\rangle & \left\langle \underline{d}_{12}, \overline{d}_{12} \right\rangle & \ldots & \left\langle \underline{d}_{1n}, \overline{d}_{1n} \right\rangle \\ A_2 & \left\langle \underline{d}_{21}, \overline{d}_{21} \right\rangle & \left\langle \underline{d}_{22}, \overline{d}_{22} \right\rangle & \ldots & \left\langle \underline{d}_{2n}, \overline{d}_{2n} \right\rangle \\ . & \ldots & \ldots & \ldots & \ldots \\ . & \ldots & \ldots & \ldots & \ldots \\ A_m & \left\langle \underline{d}_{m1}, \overline{d}_{m1} \right\rangle & \left\langle \underline{d}_{m2}, \overline{d}_{m2} \right\rangle & \ldots & \left\langle \underline{d}_{mn}, \overline{d}_{mn} \right\rangle \end{array} \quad (7)$$

Here $\left\langle \underline{d}_{ij}, \overline{d}_{ij} \right\rangle$ is the rough neutrosophic number associated to the *i*-th alternative and the *j*-th attribute.

Step 2: Determination of the relation between attributes and alternatives

The relation between attribute C_i (i = 1, 2, ..., n) and alternative B_t (t = 1, 2, ..., k) in terms of rough neutrosophic numbers is presented in the decision matrix (see the table 2).

Table 2: The relation between attributes and alternatives

$$D = \left\langle \underline{\xi}_{ij}, \overline{\xi}_{ij} \right\rangle_{n\times k} = \begin{array}{c|cccc} & B_1 & B_2 & \cdots & B_k \\ \hline C_1 & \left\langle \underline{\xi}_{11}, \overline{\xi}_{11} \right\rangle & \left\langle \underline{\xi}_{12}, \overline{\xi}_{12} \right\rangle & \ldots & \left\langle \underline{\xi}_{1k}, \overline{\xi}_{1k} \right\rangle \\ C_2 & \left\langle \underline{\xi}_{21}, \overline{\xi}_{21} \right\rangle & \left\langle \underline{\xi}_{22}, \overline{\xi}_{22} \right\rangle & \ldots & \left\langle \underline{\xi}_{2k}, \overline{\xi}_{2k} \right\rangle \\ . & \ldots & \ldots & \ldots & \ldots \\ . & \ldots & \ldots & \ldots & \ldots \\ C_n & \left\langle \underline{\xi}_{n1}, \overline{\xi}_{n1} \right\rangle & \left\langle \underline{\xi}_{n2}, \overline{\xi}_{n2} \right\rangle & \ldots & \left\langle \underline{\xi}_{nk}, \overline{\xi}_{nk} \right\rangle \end{array} \quad (8)$$

Here $\left\langle \underline{\xi}_{ij}, \overline{\xi}_{ij} \right\rangle$ is rough neutrosophic number associated with the *i*-th alternative and the *j*-th attribute.

Here $\underline{d}_{ij}, \overline{d}_{ij}$ and $\underline{\xi}_{ij}, \overline{\xi}_{ij}$ are single valued neutrosophic numbers.

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Step 3: Determination of the relation between attributes and alternatives:
Determine the rough Dice and Jaccard similarity measures $DIC_{RNS}(A, B)$ and $JAC_{RNS}(A, B)$ between the table 1 and the table 2 using equation (3) and equation (5).

Step 4: Ranking the alternatives:
Ranking of alternatives is prepared based on the descending order of rough Dice and Jaccard similarity measures. Highest value reflects the best alternative.

Step 5: End

MEDICAL DIAGNOSIS USING DICE AND JACCARD SIMILARITY MEASURE UNDER ROUGH NEUTROSOPHIC ENVIRONMENT

In medical diagnosis problem, it is necessary to collect a lot of information from modern medical technologies, which is often incomplete and indeterminate due to the complexity and ambiguity of symptoms of various diseases. Therefore, real medical diagnosis comprises of partial and incomplete information. In order to deal with the situation, rough neutrosophic set is useful.

Consider an illustrative example on medical diagnosis adopted from Pramanik and Mondal [18]. Medical diagnosis consists of a large amount of uncertainties and increased volume of information available to physicians from new updated technologies. The proposed similarity measure among the patients versus symptoms and symptoms versus diseases will provide the proper medical diagnosis. The main feature of the proposed method is that it includes rough neutrosophic set which is more flexible and easy to use.

Example: Let P = {P_1, P_2, P_3} be a set of patients, D = {Viral fever, malaria, stomach problem, chest problem} be a set of diseases and S = {Temperature, headache, stomach pain, cough, chest pain.} be a set of symptoms. The solution strategy is to examine the patient and take decision for patients for their correct disease with respect to different symptoms in rough neutrosophic environment. (see the table 3 and the table 4).

The highest rough Dice similarity measure (see the Table 5) reflects that three patients P_1, P_2, and P_3 suffer from viral fever. the proper medical diagnosis. The highest rough Jaccard similarity measure (see the Table 6) reflects that three patients P_1, P_2, and P_3 suffer from viral fever. Therefore, all three patients P_1, P_2, and P_3 suffer from viral fever. It is to be noted that rough cosine similarity measure studied By Mondal and Pramanik [18] offers the same result (see the Table 7).

ISSN 2349-4506
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Table 3: (Relation-1) The relation between Patients and Symptoms

Relation-1	Temperature	Headache	Stomach pain	cough	Chest pain
P_1	$\left\langle \begin{matrix}(0.6,0.4,0.3),\\(0.8,0.2,0.1)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.4,0.4,0.4),\\(0.6,0.2,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.5,0.3,0.2),\\(0.7,0.1,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.6,0.2,0.4),\\(0.8,0.0,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.4,0.4,0.4),\\(0.6,0.2,0.2)\end{matrix} \right\rangle$
P_2	$\left\langle \begin{matrix}(0.5,0.3,0.4),\\(0.7,0.3,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.5,0.5,0.3),\\(0.7,0.3,0.3)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.5,0.3,0.4),\\(0.7,0.1,0.4)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.5,0.3,0.3),\\(0.9,0.1,0.3)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.5,0.3,0.3),\\(0.7,0.1,0.3)\end{matrix} \right\rangle$
P_3	$\left\langle \begin{matrix}(0.6,0.4,0.4),\\(0.8,0.2,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.5,0.2,0.3),\\(0.7,0.0,0.1)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.4,0.3,0.4),\\(0.8,0.1,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.6,0.1,0.4),\\(0.8,0.1,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.5,0.3,0.3),\\(0.7,0.1,0.1)\end{matrix} \right\rangle$

Table 4: (Relation-2) The relation between symptoms and diseases

Relation-2	Viral Fever	Malaria	Stomach problem	Chest problem
Temperature	$\left\langle \begin{matrix}(0.6,0.5,0.4),\\(0.8,0.3,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.1,0.4,0.4),\\(0.5,0.2,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.3,0.4,0.4),\\(0.5,0.2,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.2,0.4,0.6),\\(0.4,0.4,0.4)\end{matrix} \right\rangle$
Headache	$\left\langle \begin{matrix}(0.5,0.3,0.4),\\(0.7,0.3,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.2,0.3,0.4),\\(0.6,0.3,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.2,0.3,0.3),\\(0.4,0.1,0.1)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.1,0.5,0.5),\\(0.5,0.3,0.3)\end{matrix} \right\rangle$
Stomach pain	$\left\langle \begin{matrix}(0.2,0.3,0.4),\\(0.4,0.3,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.1,0.4,0.4),\\(0.3,0.2,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.4,0.3,0.4),\\(0.6,0.1,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.1,0.4,0.6),\\(0.3,0.2,0.4)\end{matrix} \right\rangle$
Cough	$\left\langle \begin{matrix}(0.4,0.3,0.3),\\(0.6,0.1,0.1)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.3,0.3,0.3),\\(0.5,0.1,0.3)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.1,0.6,0.6),\\(0.3,0.4,0.4)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.5,0.3,0.4),\\(0.7,0.1,0.2)\end{matrix} \right\rangle$
Chest pain	$\left\langle \begin{matrix}(0.2,0.4,0.4),\\(0.4,0.2,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.1,0.3,0.3),\\(0.3,0.1,0.1)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.1,0.4,0.4),\\(0.3,0.2,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.4,0.4,0.4),\\(0.6,0.2,0.2)\end{matrix} \right\rangle$

Table 5: The rough Dice similarity measure between Relation-1 and Relation-2

Rough Dice similarity measure	Viral Fever	Malaria	Stomach problem	Chest problem
P_1	**0.9395**	0.8419	0.8469	0.8721
P_2	**0.9317**	0.8456	0.8204	0.8047
P_3	**0.9177**	0.8136	0.8112	0.8516

ISSN 2349-4506
Impact Factor: 2.265

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Table 6: The rough Jaccard similarity measure between Relation-1 and Relation-2

Rough Jaccard similarity measure	Viral Fever	Malaria	Stomach problem	Chest problem
P_1	**0.8903**	0.7375	0.7538	0.7943
P_2	**0.8770**	0.7202	0.7168	0.6181
P_3	**0.8525**	0.6920	0.7400	0.7561

Table 7: The rough cosine similarity measure between Relation-1 and Relation-2

Rough Cosine similarity measure	Viral Fever	Malaria	Stomach problem	Chest problem
P_1	**0.9595**	0.9114	0.8498	0.8743
P_2	**0.9624**	0.9320	0.8935	0.8307
P_3	**0.9405**	0.8873	0.8487	0.8372

CONCLUSION

In this paper, we have proposed rough neutrosophic Dice and Jaccard similarity measure and studied some of their basic properties. We have presented an application of rough neutrosophic Dice and Jaccard similarity measures in medical diagnosis. The concept presented in the paper can be applied in pattern recognition, multiple attribute decision making in rough neutrosophic environment.

REFERENCES

1. L. A. Zadeh, Fuzzy Sets. Information and Control, 8 (1965), 338-353.
2. K. Atanassov. Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20(1986), 87-96.
3. F. Smarandache, A unifying field in logics, neutrosophy: neutrosophic probability, set and logic. American Research Press, Rehoboth, 1998.
4. F. Smarandache. Linguistic paradoxes and tautologies. Libertas Mathematica, University of Texas at Arlington, IX (1999), 143-154.
5. F. Smarandache. Neutrosophic set- a generalization of intuitionistic fuzzy sets. International Journal of Pure and Applied Mathematics, 24(3) (2005), 287-297.
6. F. Smarandache. Neutrosophic set-a generalization of intuitionistic fuzzy set. Journal of Defense Resources Management, 1(1) (2010), 107-116.
7. H. Wang, F. Smarandache, Y. Q. Zhang, and R.Sunderraman. Single valued neutrosophic sets. Multispace and Multistructure, 4 (2010), 410-413.
8. S. Broumi, F. Smarandache, and M. Dhar. Rough neutrosophic sets. Italian journal of pure and applied mathematics, 32(2014), 493−502.
9. S. Broumi, F. Smarandache, and M. Dhar. Rough Neutrosophic Sets. Neutrosophic Sets and Systems, 3(2014), 60-66.

ISSN 2349-4506
Impact Factor: 2.265

Global Journal of Engineering Science and Research Management

10. P. Majumdar, and S. K. Samanta. On similarity and entropy of neutrosophic sets. Journal of Intelligent and Fuzzy Systems, 26(3) (2014), 1245-15
11. J. Ye. Vector similarity measures of simplified neutrosophic sets and their application in multicriteria decision making. International Journal of Fuzzy Systems, 16(2) (2014), 204-211.
12. S. Ye, J. Ye, Dice Similarity Measure between Single Valued Neutrosophic Multisets and Its Application in Medical Diagnosis, Neutrosophic Sets and Systems 6 (2014) 50-55.
13. J .Ye, Improved cosine similarity measures of simplified neutrosophic sets for medical diagnoses, Artificial Intelligence in Medicine, doi: 10.1016/j.artmed.2014.12.007, 2014.
14. J. Ye. Multiple attribute group decision-making method with completely unknown weights based on similarity measures under single valued neutrosophic environment. Journal of Intelligent and Fuzzy Systems, (2014), doi: 10.3233/IFS-141252
15. J. Ye, and Q. S. Zhang, Single valued neutrosophic similarity measures for multiple attribute decision making. Neutrosophic Sets and Systems 2 (2014), 48-54.
16. J. Ye. Clustering methods using distance-based similarity measures of single-valued neutrosophic sets. Journal of Intelligent Systems, (2014), doi: 10.1515/jisys-2013-0091
17. P. Biswas, S. Pramanik, and B. C. Giri, Cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers, Neutrosophic sets and Systems 8 (2015) 48-58.
18. S. Pramanik, K. Mondal, Cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. Global Journal of Advanced Research 2(1) (2015) 212-220.
19. K. Mondal, and S. Pramanik, Neutrosophic refined similarity measure based on cotangent function and its application to multi-attribute decision making, Global Journal of Advanced Research 2(2) (2015) 486-494.
20. K. Mondal, and S. Pramanik, Cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis, Journal of New Theory 4(2015) 90-102.